Gravitational Radiation and the Small-Scale Structure of Cosmic Strings

Xavier Siemens and Ken D. Olum

February 7, 2008

Institute of Cosmology

Department of Physics and Astronomy

Tufts University, Medford MA 02155, USA

Abstract

We calculate the gravitational radiation emitted by an infinite cosmic string with two oppositely moving wave-trains, in the small amplitude approximation. After comparing our result to the previously studied cases we extend the results to a new regime where the wavelengths of the opposing wave-trains are very different. We show that in this case the amount of power radiated vanishes exponentially. This means that small excitations moving in only one direction may be very long lived, and so the size of the smallest scales in a string network might be much smaller than what one would expect from gravitational back reaction. This result allows for a potential host of interesting cosmological possibilities involving ultra-high energy cosmic rays, gamma ray bursts and gravitational wave bursts.

1 Introduction

Topological defects are a prediction of most particle physics models that involve symmetry breaking and are therefore quite generic. They are formed when the topology of the vacuum manifold of the low energy theory is nontrivial [1]. The typical size of regions which acquire different vacuum expectation values depends on the type of phase transition but it can never exceed

1

the horizon size, due to causality. The specific defect formed in turn depends on the symmetry of the groups involved, in particular on the homotopy group of the vacuum manifold. If the vacuum manifold contains disconnected parts domain walls are formed, if it contains unshrinkable loops strings are formed and if it contains unshrinkable spheres monopoles are formed. More complicated hybrid defects may be formed if there is more than one phase transition. For a review see [2].

The most simple phase transition where cosmic strings are produced is

$$U(1) \xrightarrow{\eta_s} 1 \,. \tag{1}$$

The vacuum manifold is $U(1)$, which contains unshrinkable loops. The phase transition, occurring at energy η_s, traps the magnetic flux of the gauge fields associated with the $U(1)$ symmetry into strings of mass per unit length $\mu \sim \eta_s^2$. The resulting system is a network of long strings and closed loops. Numerical simulations and analytic studies show that this network quickly evolves into a "scaling regime" (see [2] and references therein), where the energy density of the string network is a constant fraction of the radiation or matter density and the statistical properties of the system such as the correlation lengths of long strings and average sizes of loops scale with the cosmic time t. The question of the size of the small-scale structure of cosmic string networks, however, has so far resisted concordance. Simulations show that most loops and the wiggles on the long strings have the smallest possible size, the simulation resolution. However, the prevailing opinion on this issue is that the size of small-scale structure also scales with the cosmic time t and its value is given by the gravitational back-reaction scale $\Gamma G \mu t$, where Γ is a number of order 100 and G is Newton's constant. This possibility was first pointed out in [3].

Cosmic strings have been a popular choice in the literature because unlike monopoles and domain walls they do not cause cosmological disasters. Indeed, they originally were considered good candidates for structure formation. Recent cosmological data, in particular the cosmic microwave background power spectrum, is not consistent with density fluctuations produced only by strings. A combination of density fluctuations produced by inflation and strings is possible, and strings with small values of μ would have little gravitational effect. Cosmic strings are also good candidates for a variety of interesting cosmological phenomena such as gamma ray bursts [4], gravitational wave bursts [5] and ultra high energy cosmic rays [6]. However, some of the predictions of these models depend sensitively on the so far unresolved question of the size of the small-scale structure.

Sakellariadou [7] computed the radiation from a cosmic string with helical standing waves, which one can consider to be composed of excitations of equal wavelength traveling in the two directions. Hindmarsh [8] calculated the radiation from colliding wiggles of different wavelengths on a straight string, in a low-amplitude approximation, and found that the radiation rate from small wavelength wiggles approached a constant as the wavelength of the wiggles in the opposite direction became large. However, Garfinkle and Vachaspati [9] showed that a wiggle moving on a straight string does not radiate. This poses a puzzle because an excitation of sufficiently large wavelength would seem to appear straight to a tiny wiggle passing over it.

With this work we hope to shed some light on these issues. The following section contains a review of string motion and a derivation of the energy emitted in the form of gravitational radiation per unit solid angle. Section 3 starts with a derivation of the power emitted from infinite planar excitations on a string with a subsequent specialisation to sinusoidal wave-trains in the case where the amplitude is small. We show that in the similar wavelength case the result of Hindmarsh [8] applies and how an argument on the limit on the size of the small-scale structure consistent with [3] can be obtained from this result. We then demonstrate that this argument cannot generally be made and in particular show that when the wavelengths are very different the radiation vanishes exponentially. In section 4, we discuss these results and their relevance to the question of the small-scale structure in cosmic string networks and conclude with some remarks on the effect of these results on cosmological phenomena.

2   Preliminaries

When the typical length scale of a cosmic string is much larger than its thickness, $\delta_s \sim \eta_s^{-1}$, and long-range interactions between different string segments can be neglected, the string can be accurately modeled by a one dimensional object. Such an object sweeps out a two dimensional surface in space-time referred to as the string world-sheet.

The infinitesimal line element in Minkowski space-time with metric $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ is

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu = \eta_{\mu\nu}x^\mu_{,a}x^\nu_{,b}d\zeta^a d\zeta^b, \qquad (2)$$

where $a = 0, 1$ labels the two internal parameters of the string world-sheet

and $x^\mu_{,a} = \partial x^\mu / \partial \zeta^a$. One can then write the induced metric on the world-sheet of the string as

$$\gamma_{ab} = \eta_{\mu\nu} x^\mu_{,a} x^\nu_{,b}. \tag{3}$$

For an infinitely thin string we can use the Nambu-Goto action. It is proportional to the invariant area swept by the string,

$$S = -\mu \int dA = -\mu \int d^2\zeta \sqrt{-\gamma}, \tag{4}$$

where $\gamma = \det(\gamma_{ab})$ and μ is the mass per unit length of the string. The parametrisation-invariant energy-momentum tensor for a cosmic string is given by

$$T^{\mu\nu}(x) = \mu \int d^2\zeta \sqrt{-\gamma} \gamma^{ab} x^\mu_{,a} x^\nu_{,b} \delta^{(4)}(x - X(\zeta)). \tag{5}$$

In the light-cone gauge the metric takes the form

$$\gamma_{ab} = \sqrt{-\gamma} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \gamma^{ab} = \frac{1}{\sqrt{-\gamma}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{6}$$

because the arc element is $ds^2 = 2x_{,u} \cdot x_{,v} du dv$ with $x^\mu = x^\mu_R(u) + x^\mu_L(v)$, and $x'^\mu_R(u)$ and $x'^\mu_L(v)$ are both null vectors. In this gauge the stress energy tensor (5) can be put in the form

$$T^{\mu\nu}(x) = \mu \int du dv (x^\mu_{,u} x^\nu_{,v} + x^\nu_{,u} x^\mu_{,v}) \delta^{(4)}(x - x(\zeta)). \tag{7}$$

Far from a source localized in space and time, the total energy radiated in the form of gravity waves in the direction of $\mathbf{k}$ is [10]

$$\frac{dE}{d\Omega} = 2G \int_0^\infty d\omega \omega^2 \{ T^{\mu\nu*}(k) T_{\mu\nu}(k) - \frac{1}{2} \left| T^\mu_\mu(k) \right|^2 \}. \tag{8}$$

So what we need is the Fourier transform of the stress energy tensor

$$T^{\mu\nu}(k) = \frac{1}{2\pi} \int d^4x T^{\mu\nu}(x) e^{ik \cdot x} \tag{9}$$

which is given by

$$T^{\mu\nu}(k) = \frac{1}{2\pi} \mu \int d^4x e^{ik \cdot x} \int du dv (x^\mu_{,u} x^\nu_{,v} + x^\nu_{,u} x^\mu_{,v}) \delta^{(4)}(x - x(\zeta)). \tag{10}$$
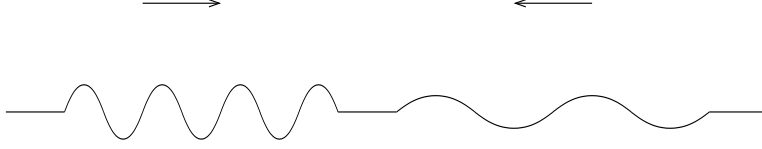
4

Figure 1: Two oppositely moving wave-trains on an infinite straight cosmic string. We ignore the effects of the kinks at the edges of the wave-trains because later we will be taking the length of the wave-trains to ∞.

Using $x^\mu = [a^\mu(u) + b^\mu(v)]/2$ yields for (10),

$$T^{\mu\nu}(k) = \frac{1}{8\pi}\mu \int dudv(a'^\mu b'^\nu + a'^\nu b'^\mu)e^{ik\cdot(a+b)/2}. \tag{11}$$

Following [8] this expression can in turn can be written as

$$T^{\mu\nu}(k) = \frac{1}{8\pi}\mu(A^\mu(k)B^\nu(k) + A^\nu(k)B^\mu(k)) \tag{12}$$

where

$$A^\mu(k) = \int_{-\infty}^{+\infty} d\xi a'^\mu(\xi)e^{ik\cdot a(\xi)/2}, \; B^\mu(k) = \int_{-\infty}^{+\infty} d\xi b'^\mu(\xi)e^{ik\cdot b(\xi)/2}. \tag{13}$$

We can then re-write the total energy in the direction of $\mathbf{k}$ (8) as

$$\frac{dE}{d\Omega} = \frac{1}{16\pi^2}G\mu^2 \int_0^\infty d\omega\omega^2\{|A|^2|B|^2 + |A^*\cdot B|^2 - |A\cdot B|^2\}. \tag{14}$$

3   Gravitational Radiation from Cosmic Strings

3.1   Planar Excitations

We consider planar string excitations moving on the z-axis in the two directions as shown in Figure 1.

We are free to take

$$a'^\mu(u) = (1, f'(u), 0, -\sqrt{1-f'^2}), \; a^\mu(u) = \left(u, f(u), 0, -\int \sqrt{1-f'^2}du\right) \tag{15}$$

where the sign of the square root is chosen such that u decreases in the positive z direction, which yields from (13)

$$A^\mu(k) = \int_{-\infty}^\infty du\, a'^\mu(u)\exp\frac{i}{2}\left[\omega u - k_x f + k_z \int \sqrt{1-f'^2}du\right] \tag{16}$$

5

exactly. The situation for $B^\mu(k)$ is completely analogous.

Equation (14) gives the energy radiated from a localized interaction. In Appendix A, we extend this to the power per unit length from an infinite periodic wave train. We find that the radiation is emitted in a discrete set of cones and at a discrete set of frequencies. Specifically, let Δ_a be the average of $|a'_z|$,

$$\Delta_a = \frac{1}{\lambda_a} \int_0^{\lambda_a} du \sqrt{1 - f'^2} \tag{17}$$

where λ_a is the wavelength of the wiggles of $a^\mu(u)$, and let

$$\kappa_a = \frac{2\pi}{\lambda_a} \tag{18}$$

and similarly for Δ_b and κ_b. Then for each set of positive integers n, m satisfying

$$(1 - \Delta_a)/(1 + \Delta_b) < |n\kappa_a/m\kappa_b| < (1 + \Delta_a)/(1 - \Delta_b). \tag{19}$$

we have radiation with polar angle θ given by

$$\cos\theta = \frac{n\kappa_a - m\kappa_b}{n\kappa_a\Delta_b + m\kappa_b\Delta_a}, \tag{20}$$

at frequency

$$\omega = 2\frac{n\kappa_a\Delta_b + m\kappa_b\Delta_a}{\Delta_a + \Delta_b}, \tag{21}$$

(so that $\omega + \Delta_a k_z = 2n\kappa_a$ and $\omega - \Delta_b k_z = 2m\kappa_b$) with power per unit length per unit azimuthal angle

$$\frac{dP}{dzd\phi} = \frac{G\mu^2}{\Delta_a + \Delta_b} \sum_{n,m}(n\kappa_a + m\kappa_b)|A_n|^2|B_m|^2 \tag{22}$$

where

$$A_n^\mu = \frac{1}{\lambda_a} \int_{-\lambda_a/2}^{\lambda_a/2} du\, a'^\mu e^{ik\cdot a(u)/2}$$

$$= \frac{1}{\lambda_a} \int_{-\lambda_a/2}^{\lambda_a/2} du\, a'^\mu \exp\frac{i}{2}\left[\omega u + k_z \int_0^u \sqrt{1 - f'^2}du' - k_x f(u)\right] \tag{23}$$

If we fix the shape of a^μ, but vary the scale so that λ_a becomes large, then Δ_a is fixed and $\kappa_a \to 0$. If we follow a single mode n, m as we go toward this limit, θ reaches π at a finite value of κ_a, given by

$$\kappa_a^m = \frac{m}{n}\kappa_b(1 - \Delta_a)/(1 + \Delta_b). \tag{24}$$

For larger values of κ_a, the given mode does not exist.

3.2 Sinusoidal waves

3.2.1 Small amplitudes

We will consider excitations whose amplitude is small compared to their wavelength, so that $f' \ll 1$. This enables us to replace $\sqrt{1 - f'^2}$ by its average, Δ_a, in (23), to get

$$A_n^\mu \approx \frac{1}{\lambda_a} \int_{-\lambda_a/2}^{\lambda_a/2} du \, a'^\mu(u) \exp i \left[n\kappa_a u - k_x f(u)/2 \right]. \tag{25}$$

In particular if we take the wave trains to be sinusoidal, namely,

$$f'(u) = \epsilon_a \cos(\kappa_a u), \; f(u) = \frac{\epsilon_a}{\kappa_a} \sin(\kappa_a u) \tag{26}$$

with $\epsilon_a \ll 1$, and thus

$$\Delta_a \approx 1 - \epsilon_a^2/4, \tag{27}$$

for the time component we have

$$A_n^0 \approx \frac{1}{\lambda_a} \int_{-\lambda_a/2}^{\lambda_a/2} du \, \exp i \left[n\kappa_a u - \epsilon_a k_x \sin(\kappa_a u)/2\kappa_a \right] \tag{28}$$

which can be integrated to get

$$A_n^0 \approx J_n \left(\frac{\epsilon_a k_x}{2\kappa_a} \right) \tag{29}$$

where J_n is the Bessel function of order n. The first component can be put in terms of A_0 using integration by parts

$$A_n^1(k) = -\frac{2n\kappa_a}{k_x} A_n^0 \tag{30}$$

and the third component can be also be written in terms of A_0 as

$$A_n^3 = \Delta_a A_n^0 \tag{31}$$

so that

$$|A_n|^2 \approx \left(1 - \frac{4n^2\kappa_a^2}{k_x^2} - \Delta_a^2 \right) J_n^2 \left(\frac{\epsilon_a k_x}{2\kappa_a} \right). \tag{32}$$
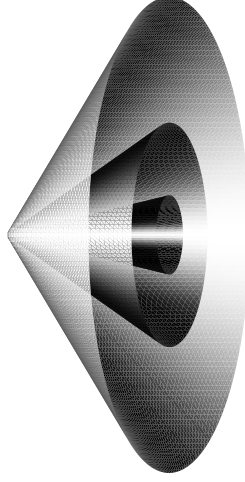
Figure 2: Discrete radiation cones given by (20) and (33) for $m = 1$ and $n = 2, 4$ and 20 with $\epsilon_a^2 = \epsilon_b^2 = 0.1$ and $\kappa_a = \kappa_b$. Shading denotes the intensity of the radiation, with white being most intense. As n gets larger, the cone becomes narrow and the radiation gets concentrated in a narrow band around where the cone intersects the plane of the string.

A similar analysis for B and using (27) yields the expression for the power in terms of Bessel functions,

$$\frac{dP}{dzd\phi} \approx \frac{G\mu^2}{\Delta_a + \Delta_b} \sum_{n,m} (n\kappa_a + m\kappa_b) \left(\frac{\epsilon_a^2}{2} - \frac{4n^2\kappa_a^2}{k_x^2} \right) \left(\frac{\epsilon_b^2}{2} - \frac{4m^2\kappa_b^2}{k_x^2} \right)$$
$$\times J_n^2 \left(\frac{\epsilon_a k_x}{2\kappa_a} \right) J_m^2 \left(\frac{\epsilon_b k_x}{2\kappa_b} \right) \tag{33}$$

As an example, Figure 2 shows some of the discrete radiation cones given by (20) with power calculated from (33).

3.2.2 Similar wavelength wiggles and the back-reaction argument

If κ_a and κ_b are similar in magnitude, and $\epsilon_a, \epsilon_b \ll 1$, then the arguments of both Bessel functions will be much less than 1. In this case we can use the small-argument approximation for the Bessel functions [11],

$$J_n(x) \approx \frac{x^n}{2^n n!} \tag{34}$$

and so we see that the dominant contribution to (33) comes from the $n = m = 1$ mode, and the power can be written

$$\frac{dP}{dzd\phi} \approx \frac{G\mu^2\epsilon_a^2\epsilon_b^2}{32}(\kappa_a + \kappa_b).$$ (35)

Integrating over ϕ yields

$$\frac{dP}{dz} \approx \frac{1}{16}\pi G\mu^2\epsilon_a^2\epsilon_b^2(\kappa_a + \kappa_b)$$ (36)

This agrees with the result of [8]. The factor of 16 results from our choice of normalization in the definitions of $A^\mu(k)$ and $B^\mu(k)$.

An argument that can be thought to follow from this result is the following. Consider short wavelength excitations in b interacting with long wavelength excitations in a. If we put

$$\kappa_a \ll \kappa_b$$ (37)

in (36) we obtain

$$\frac{dP}{dz} \sim \frac{1}{8}\pi^2 G\mu^2\epsilon_a^2\epsilon_b^2\frac{1}{\lambda_b} = \frac{1}{8}\pi^2 G\mu\epsilon_a^2\frac{\delta\mu}{\lambda_b}$$ (38)

where $\delta\mu \sim \epsilon_b^2\mu$ is the contribution of the short wavelength wiggles to the mass per unit length of the string. We expect the radiated energy to be taken from the small-scale wiggles, so we expect

$$\frac{d(\delta\mu)}{dt} \sim -\frac{dP}{dz} \sim -\frac{1}{8}\pi^2 G\mu\epsilon_a^2\frac{\delta\mu}{\lambda_b}$$ (39)

which has the solution

$$\delta\mu \propto e^{-t/\tau}$$ (40)

with

$$\tau = \frac{8\lambda_b}{\pi^2\epsilon_a^2 G\mu}.$$ (41)

If we assume that large wavelength wiggles exist with $\epsilon \sim 1$, we would conclude by the above argument that the minimum wavelength of wiggle that can survive until the present day is

$$\lambda_{\min} \sim G\mu t_0$$ (42)

where t_0 is the present age of the universe. On smaller scales, wiggles are exponentially suppressed.

9

If we extend this analysis to a loop, the structure of the loop that enables it to be closed must involve features whose amplitude is comparable to their wavelength, so the condition $\epsilon \sim 1$ is always met.

However, it turns out that (36) is never correct in the regime of (37) with $\epsilon_a \sim 1$. Even $\epsilon_a, \epsilon_b \ll 1$ is not sufficient, because we might have, for example, $k_x/\kappa_a \gg 1$, and then the argument of J_n in (33) would not be small. Explicitly,

$$
\begin{aligned}
k_x &= \omega \sin(\theta) \cos(\phi) \\
&= \frac{\cos \phi}{\Delta_a + \Delta_b} \sqrt{(\Delta_b n \kappa_a + \Delta_a m \kappa_b)^2 - (n \kappa_a - m \kappa_b)^2}
\end{aligned}
\tag{43}
$$

so k_x is at most of order $\sqrt{nm\kappa_a\kappa_b}$ and thus we are in the small-argument regime whenever

$$
\epsilon_a^2 \kappa_b/\kappa_a \ll 1, \ \epsilon_b^2 \kappa_a/\kappa_b \ll 1.
\tag{44}
$$

Larger values of n and m could in principle make the arguments of the Bessel functions larger, but it turns out that in this situation the contribution is dominated by that of the lowest mode.

One can write (44) in terms of the wavelengths of the sinusoids, $\lambda_{a,b} = 2\pi/\kappa_{a,b}$ and their amplitudes, $A_{a,b} = \epsilon_{a,b}/\kappa_{a,b}$, as

$$
A_a, A_b \ll \sqrt{\lambda_a \lambda_b}
\tag{45}
$$

which is invariant under Lorenz boosts along the string. Rather than merely requiring that the amplitude of each wave train be small compared to its own wavelength to have the simple case, we must require that each amplitude be small as compared to the geometric mean wavelength. Note that these conditions cannot hold if $\lambda_a \gg \lambda_b$ and $\epsilon_a \sim 1$. As an example of this, in the case of small "chiral" wiggles (those moving in one direction only) on a loop, the conditions (44) don't hold and we expect wiggles to survive on the loop longer than (41).

It should be noted that if both of the wiggles have about the same wavelengths then an argument analogous to the one above can be made [12]. In this case wiggles in both directions contribute to the radiation power, and the extra energy in the string declines as $1/t$.

3.2.3 Chiral excitations

In the following we take the small argument limit for the B part of (33), namely $\epsilon_b^2 \kappa_a/\kappa_b \ll 1$, or $A_b \ll \sqrt{\lambda_a \lambda_b}$, but do not use that approximation

for the A part of the expression. This corresponds to the situation where the opposing excitations have very different wavelengths. Thus (33) becomes

$$\frac{dP}{dzd\phi} = \frac{G\mu^2\epsilon_b^2}{8} \sum_n (n\kappa_a + \kappa_b) \left(\frac{4n^2\kappa_a^2}{k_x^2} - \frac{\epsilon_a^2}{2} \right) J_n^2 \left(\frac{\epsilon_a k_x}{2\kappa_a} \right). \qquad (46)$$

For $m = 1$, the bounds (19) become

$$\frac{(1 - \Delta_a)\kappa_b}{(1 + \Delta_b)\kappa_a} < n < \frac{(1 + \Delta_a)\kappa_b}{(1 - \Delta_b)\kappa_a} \qquad (47)$$

which using (27) can be written as

$$\frac{x}{8} < n < \frac{8x}{\epsilon_a^2 \epsilon_b^2} \qquad (48)$$

where

$$x = \frac{\epsilon_a^2 \kappa_b}{\kappa_a}. \qquad (49)$$

Let us first consider what happens to the $n = 1$ mode as κ_a approaches the limit in (24). In this limit, θ becomes π, while ω and κ_a approach fixed, finite values. Thus $k_x/\kappa_a \to 0$, we can take the small argument limit of the Bessel function and the power in this mode approaches a finite value given by (36). When κ_a exceeds the limit of (24), the $n = 1$ mode vanishes suddenly, causing a discontinuity in the emitted power as a function of κ_a. Similarly, if we let ϵ_a approach the limit given by $\epsilon_a^2 \approx 8\kappa_a/\kappa_b$, we see that the power in the $n = 1$ mode is again given by (36), which increases with ϵ_a^2 until it suddenly vanishes.

As κ_a decreases, the modes successively vanish (although only the energy in the $n = 1$ mode does so discontinuously), but the remaining modes that are far from the limiting wave number grow larger. To see the overall effect, we consider the case where $\epsilon_a^2\kappa_b/\kappa_a \gg 1$, or in terms of the amplitude $A_a \gg \sqrt{\lambda_a\lambda_b}$, so that many different modes contribute to the total radiation.

In Appendix B we use the large-order approximation for Bessel functions to compute the power (46) in this regime. We find that the power declines exponentially with x,

$$\frac{dP}{dz} \sim G\mu^2\epsilon_b^2\kappa_a e^{-\alpha x} \qquad (50)$$

where $\alpha \approx 0.07867$. Thus there is essentially no radiation from wiggles of significant amplitude and very different wavelengths.

11

To compare the analytic approximation (50) with our original expression (46), in Figure 3 we show a plot of $\frac{dP}{dz}$ versus $x = \epsilon_a^2 \kappa_b / \kappa_a$ with $\epsilon_a = .1$, $\epsilon_b = .05$ and κ_a fixed. The dashed line is

$$\ln \left(\frac{1}{G\mu^2 \kappa_a} \frac{dP}{dz} \right) = \alpha x + \beta \tag{51}$$

with a best-fit value $\beta = -5.69$, and $\alpha = -0.07867$ as above. The points are given by the sum over n and numerical integration over ϕ of (46). In the large x regime, we see that the agreement with the analytic approximation is excellent.

The exponential suppression we have just found is valid when the Lorentz invariant conditions

$$A_b \ll \sqrt{\lambda_a \lambda_b}, A_a \gg \sqrt{\lambda_a \lambda_b} \tag{52}$$

are satisfied. It is interesting to consider the effect of a Lorentz boost that makes both wiggle wavelengths similar. This boost makes the condition (52) for A_a read $\epsilon_a \gg 1$. In this case we could not have performed the calculation at all because this is not the regime in which (25) is a valid approximation. It must be true, however, because of Lorentz invariance, that the radiation is also exponentially suppressed. Although at first glance this may seem puzzling, it is not too difficult to understand this result: In this limit, even though the wavelengths are similar, the amplitude of one of the wave trains is much larger than the other ($A_a \gg A_b$) so that the larger wave-train appears to be traveling on an almost straight string and would not be expected to radiate [9].

4   Discussion and Conclusions

We have calculated the power radiated in the form of gravitational waves from oppositely moving excitation wave-trains in the small amplitude regime when the wavelengths are comparable and when they are very different. We have shown that small opposing excitations on strings will only radiate significantly if their wavelengths are comparable. In this case we can always use (36) for the power and the extra energy in the form of wiggles decreases as $1/t$ as discussed in the literature [12].

If, on the other hand, the wavelengths are very different then the gravitational radiation can be suppressed. If the amplitude A_a of the long-wavelength excitations is greater than the geometric mean wavelength, $\sqrt{\lambda_a \lambda_b}$,
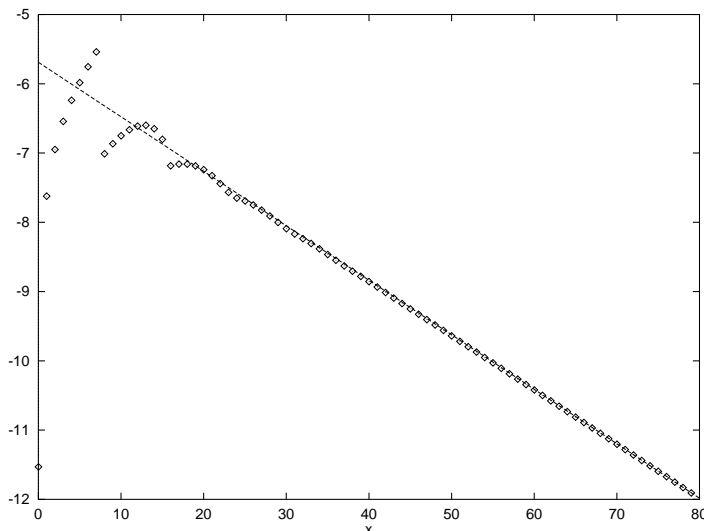
12

Figure 3: Plot of $\ln\left(\frac{1}{G\mu^2\kappa_a}\frac{dP}{dz}\right)$ vs. $x = \epsilon_a^2\kappa_b/\kappa_a$ comparing our analytic approximation (50,51) with $\beta = -5.69$ and $\alpha = -0.07867$ (dashed line) with (46) calculated numerically for $\epsilon_a = .1$, $\epsilon_b = .05$ (points).

then the radiation rate is exponentially small. Thus unless the amplitude of the long-wavelength excitations is very small compared to their wavelength, there is little interaction between modes of very different wavelengths.

The situation where the two wavelengths are very different occurs in loops with chiral wiggles, i.e., when the wiggles are only on the right- or left-moving excitations. In this case because the radiation vanishes exponentially the wiggles may be very long-lived. We therefore expect that if the loop is originally in a stable (non self-intersecting) trajectory it will radiate and shrink until it self-intersects, at the latest when the string loop is the same size as the wiggles living on it. This may take a long time because the tension on the strings is reduced by the presence of wiggles and the motion may be slow [13].

When an excitation travels on an infinite string, it will eventually be exposed to every possible sort of excitation going the other way, and will thus eventually lose its energy by gravitational radiation. However, when an excitation travels on a loop, it will encounter the same oppositely moving excitations over and over again. It is possible that when a loop is formed there will be, by chance, an excess of (say) right-going energy over left-going energy over a broad range of wavelengths. In this case, energy will be radiated until the

13

left-going excitations have been eliminated and some energy still remains in the right-going ones. Thus the string may become chiral with respect to a certain range of wavelengths, and the chiral wiggles in this range may then become long-lived. This process is quite complicated, and we will not try to analyze it here. We merely note that certain initial conditions may lead to long-lived chiral wiggles over certain wavelength ranges, and we briefly investigate the consequences of such excitations.

Almost all proposals for cosmic-string-induced astrophysical phenomena are related to cusps. These are regions of the string that achieve extremely high Lorentz boosts [2]. Therefore here we will focus on the effects that the existence of small wiggles may have on cusps.

Near a wiggly cusp the effective mass per unit length is

$$\mu_{\text{eff}} = \mu(1 + \gamma^2 \epsilon^2) \tag{53}$$

where ϵ is the amplitude to wavelength ratio of the wiggles and γ is the (transverse) Lorentz boost. If we think of the string as having a thickness due to the wiggles then before the cusp can be formed the wiggles will typically overlap when the gamma factor reaches the value [14]

$$\gamma_o \sim \sqrt{L/d} \tag{54}$$

where L is the size of the loop on which the cusp appears, $d \sim \epsilon/\kappa$ is the effective thickness due to the wiggles and $\kappa \sim 1/\lambda$ is the wiggle wavenumber. The cusp may also not form at all if there is sufficient back-reaction from the wiggles to deviate from regular Nambu-Goto motion of the string, namely when the gamma factor in (53) reaches the value

$$\gamma_b \sim 1/\epsilon. \tag{55}$$

Which of the two processes is the dominant one depends on which one of the two gamma factors is the smallest, since it will be the one reached first by the cusp.

It follows from this that there are two distinct wiggliness regimes. If $\gamma_b > \gamma_o$, or $L\kappa\epsilon < 1$, there is not much change in the effective mass of the string and the wiggles will not significantly affect the string motion. However, before the cusp can be formed, because of the overlap of the wiggles, we will typically have a self-intersection that chops off a loop of size $l \sim \sqrt{\epsilon L/\kappa}$ which in turn will fragment into about $\sqrt{\kappa L/\epsilon}$ loops of the wiggle size ϵ/κ.
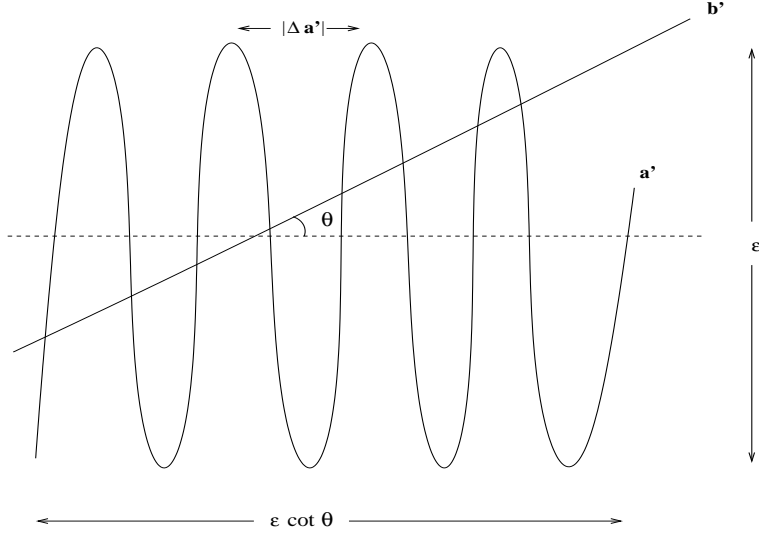
14

Figure 4: Intersection on the unit sphere of a wiggly $\mathbf{a}'$ and a straight $\mathbf{b}'$. The dotted line resulting from averaging out the wiggles in $\mathbf{a}'$ makes an angle θ with $\mathbf{b}'$.

If, on the other hand, $\gamma_o > \gamma_b$, or $L\kappa\epsilon > 1$, we are in the situation where the back-reaction is important. Much work has gone into understanding the behaviour of strings in this regime [15]. Typically one averages the string over scales larger than the wiggle size and the effect is to increase the effective mass density and decrease the tension of the string. The situation is then remarkably similar to that of a superconducting string with a chiral neutral current and we expect the cusp in this case to be smoothed out and self-intersections to occur near it some of the time as discussed in [16].

We can try to estimate the number of cusps formed in both cases. If we consider the section of the unit sphere where a wiggly $\mathbf{a}'$ and a straight $\mathbf{b}'$ intersect, as in Figure 4, we can see that the number of crossings must be

$$N_c \sim \epsilon \cot\theta / |\Delta\mathbf{a}'| \qquad (56)$$

where $\epsilon \cot\theta$ is the length available for crossings in the section where the $\mathbf{a}'$ and $\mathbf{b}'$ intersect on the unit sphere and $|\Delta\mathbf{a}'|$ is the distance between peaks on the unit sphere. Typically

$$|\Delta\mathbf{a}'| \sim |\mathbf{a}''|\Delta\sigma \sim \lambda/L \qquad (57)$$

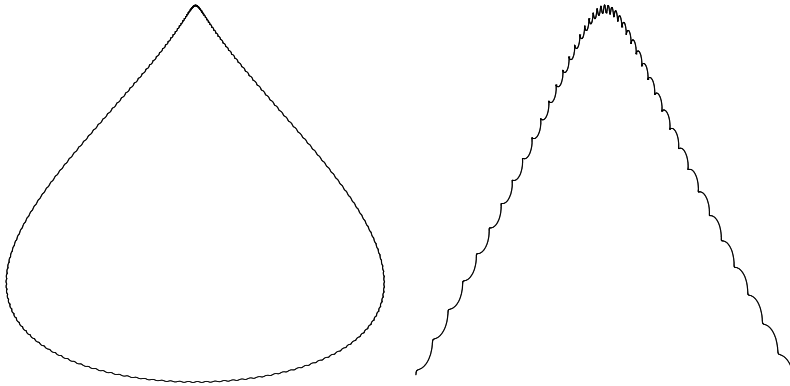where λ is the wavelength of the wiggles and L is the size of the loop. Taking

15

Figure 5: A "cusp" in a wiggly string with $L\kappa\epsilon > 1$. The wiggles have led the overall shape of the string to be smoothed out. Instead of one large cusp, we expect a large number of small cusps.

$\cot\theta \sim 1$ gives

$$N_c \sim L\kappa\epsilon \qquad (58)$$

for the number of cusps. It is interesting to note that this is also the quantity that determines whether we are in the overlap or back-reaction regime. This means that in the overlap case there will still be one cusp, just as in the case where there are no wiggles. As we have argued, however, self-intersections due to overlap will take place before the cusp can be fully formed. In the back-reaction dominated case instead of a large cusp we will have about $L\kappa\epsilon$ small cusps on the smoothed out curve. In Figures 5 and 6 we plot pictures of Burden loops onto which we have superimposed chiral helical wiggles in the $L\kappa\epsilon > 1$ case.

From this analysis it appears that wiggly cosmic string cusps may solve some of the problems currently faced by cosmic-string-induced astrophysical phenomena. Wiggly cusps could produce substantially more ultra-high energy cosmic rays because of the presence of many small cusps. Gamma ray bursts may not be repeated because of self-intersections near the cusp. Gravitational wave bursts may be more frequent since, like cosmic rays, most of the energy comes from a small area around the cusp. Finally, wiggles may also lead to fine structure in gravitational wave and gamma ray bursts.

Although some of these require different wiggliness regimes and would appear mutually exclusive in fact they are not; we can "have it both ways" because as we have shown modes of different wavelengths barely interact and as a consequence chirality can be independently established at different
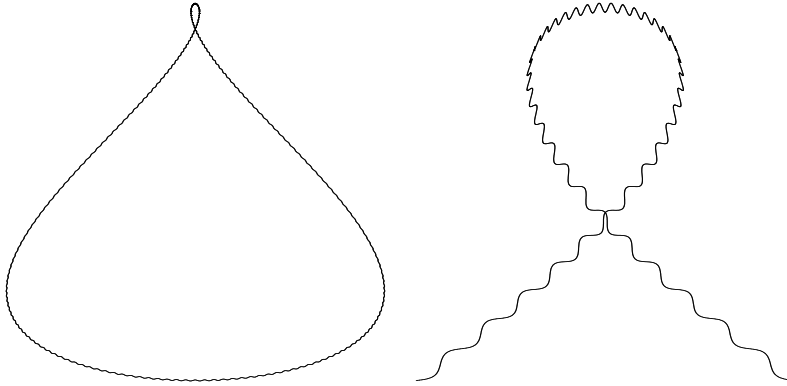
Figure 6: Another possibility for a "cusp" in a wiggly string with $L\kappa\epsilon > 1$. In this case the smoothing effect of the wiggles has led to a self-intersection.

wavelengths.

5 Acknowledgments

A   Periodic sources

In this appendix, we go from (14), which gives the energy radiated from a localized interaction, to the power per unit length for an infinite periodic source. To do this, we consider a wave train of length L in a and another of length l in b, and then let L and l go to infinity. The resulting energy will be proportional to Ll, as shown below. We will ignore any contributions coming from the ends of the wave trains, because these will not be proportional to Ll.

Thus we will take f to be an odd periodic function in u, for N_a periods of length λ_a (with $L = N_a\lambda_a$) centered around $u = 0$, and $f = 0$ outside this range. Since f is odd, all components of A^μ will be real and (14) becomes

$$\frac{dE}{d\Omega} = \frac{G\mu^2}{16\pi^2} \int_0^\infty d\omega\omega^2 \left|A\right|^2 \left|B\right|^2 . \qquad (59)$$

Because we are interested only in terms proportional to Ll, we exclude the contribution of the straight parts of the string to the integral in (16), and write this equation as

$$A^\mu = \sum_{j=-(N_a-1)/2}^{(N_a-1)/2} \int_{(j-1/2)\lambda_a}^{(j+1/2)\lambda_a} du\, a'^\mu e^{ik\cdot a(u)/2}\,. \tag{60}$$

Now a' is periodic, and so $a(u+j\lambda_a) - a(u)$ is a constant independent of u. Thus

$$A^\mu = \sum_{j=-(N_a-1)/2}^{(N_a-1)/2} e^{ijK_+\lambda_a/2} \int_{-\lambda_a/2}^{\lambda_a/2} du\, a'^\mu e^{ik\cdot a(u)/2} \tag{61}$$

where

$$K_+ = \frac{k\cdot(a(\lambda_a) - a(0))}{\lambda_a} = \frac{1}{\lambda_a}\int_0^{\lambda_a} du\, k\cdot a'(u)$$
$$= \omega + k_z\Delta_a = \omega(1 + \Delta_a\cos\theta) \tag{62}$$

with

$$\Delta_a = \frac{1}{\lambda_a}\int_0^{\lambda_a} du\,\sqrt{1 - f'^2}. \tag{63}$$

The sum in (61) is

$$\sum_{j=-(N_a-1)/2}^{(N_a-1)/2} e^{ijx} = \frac{\sin(N_a x_a/2)}{\sin(x_a/2)} \tag{64}$$

with $x_a = \lambda_a K_+/2$. This function has peaks of height N_a whenever x_a is an integer multiple of 2π and in the vicinity of these peaks the function is effectively $N_a\,\mathrm{sinc}(N_a x_a/2\pi)$. However, because what appears in (59) is the square of $A^\mu(k)$ what we actually need to evaluate is $N_a^2\,\mathrm{sinc}^2(N_a x_a/2\pi)$. In the large N_a limit $N_a\,\mathrm{sinc}^2(N_a y)\to\delta(y)$ so we can write its square as

$$\frac{\sin^2(N_a x/2)}{\sin^2(x/2)} \approx N_a \sum_{n=-\infty}^{\infty} \delta(\frac{\lambda_a K_+}{2(2\pi)} - n) = 2\kappa_a N_a \sum_{n=-\infty}^{\infty} \delta(K_+ - 2n\kappa_a). \tag{65}$$

A similar set of expressions can be written for $B^\mu(k)$ which allow (59) to be re-written as

$$\frac{dE}{d\omega d\Omega} = \frac{G\mu^2}{4\pi^2}\kappa_a\kappa_b N_a N_b \sum_{n,m} \omega^2\delta(K_+ - 2n\kappa_a)\delta(K_- - 2m\kappa_b)\mathcal{A}^2\mathcal{B}^2 \tag{66}$$

18

where

$$\mathcal{A} = \int_{-\lambda_a/2}^{\lambda_a/2} du\, a'^\mu e^{ik\cdot a(u)/2}, \ \mathcal{B} = \int_{-\lambda_b/2}^{\lambda_b/2} dv\, b'^\mu e^{ik\cdot b(v)/2}. \qquad (67)$$

All the variables with the b subscript have the obvious meaning and $K_- = \omega(1 - \Delta_b \cos\theta)$. The sum in (66) is performed for positive values of n and m only since K_+ and K_- are nonnegative.

We can now use the δ functions to perform the integral in (59) over ω and to integrate over θ. We do this by a suitable change of variables, namely,

$$\omega = (\Delta_b K_+ + \Delta_a K_-)/(\Delta_a + \Delta_b), \cos\theta = (K_+ - K_-)/(\Delta_b K_+ + \Delta_a K_-)) \ (68)$$

so that

$$d\omega\, d\cos\theta = dK_+ dK_-/(\Delta_b K_+ + \Delta_a K_-) = dK_+ dK_-/(\omega(\Delta_a + \Delta_b)). \quad (69)$$

Upon integration we get

$$\frac{dE}{d\phi} = \frac{G\mu^2}{\Delta_a + \Delta_b} Ll \sum_{n,m} \omega |A_n|^2 |B_m|^2 \qquad (70)$$

where the vector

$$A_n^\mu = \frac{1}{\lambda_a} \int_{-\lambda_a/2}^{\lambda_a/2} du\, a'^\mu(u) e^{ik\cdot a(u)/2} \,. \qquad (71)$$

and there is a similar expression for B_n^μ. Dividing (70) by the volume of the world sheet where the interaction occurs, $Ll/2$, yields the power per unit length per unit azimuthal angle ϕ

$$\frac{dP}{dz\, d\phi} = \frac{G\mu^2}{\Delta_a + \Delta_b} \sum_{n,m} (n\kappa_a + m\kappa_b) |A_n|^2 |B_m|^2 \qquad (72)$$

Keeping the above in mind we can see that the significance of the appearance of the δ functions is that for periodic sources the radiation is only emitted in discretized directions and frequencies. Explicitly, for a set of given Δ_a, Δ_b, κ_a and κ_b we have that

$$\omega = 2\frac{n\kappa_a \Delta_b + m\kappa_b \Delta_a}{\Delta_a + \Delta_b} \,, \qquad (73)$$

$$\cos\theta = \frac{n\kappa_a - m\kappa_b}{n\kappa_a \Delta_b + m\kappa_b \Delta_a} \,. \qquad (74)$$

19

This discreteness in frequencies and angles arises from the temporal and spatial periodicity of the system.

If we had $\Delta_a = \Delta_b = 1$, then the right hand side of (74) would always lie between -1 and 1. However, for waves of finite amplitude, this is not the case and thus not all modes are present. This limitation restricts the sum over n and m to satisfy

$$(1 - \Delta_a)/(1 + \Delta_b) < |n\kappa_a/m\kappa_b| < (1 + \Delta_a)/(1 - \Delta_b). \qquad (75)$$

B   Large order approximation

In this appendix, we compute an analytic approximation to the radiation power (46) using the Bessel function approximation for large order [11]

$$J_n(n \operatorname{sech} \alpha) \approx \frac{e^{n(\tanh \alpha - \alpha)}}{\sqrt{2\pi n \tanh \alpha}} \qquad (76)$$

which is valid when $\tanh \alpha > 1/n$.

Using (43) we can see that

$$
\begin{aligned}
\frac{k_x^2}{\kappa_a^2} &= \frac{\cos^2(\phi)}{4} \left\{ (\Delta_b n + \Delta_a x/\epsilon_a^2)^2 - (n - x/\epsilon_a^2)^2 \right\} \\
&\approx \frac{\cos^2(\phi)}{4} \left\{ 4nx/\epsilon_a^2 - \frac{x^2}{2\epsilon_a^2} - \frac{\epsilon_b^2}{2} n^2 \right\}
\end{aligned}
\qquad (77)
$$

which allows us to express the square of the argument of the remaining Bessel function in (46) as

$$\frac{k_x^2 \epsilon_a^2}{4\kappa_a^2} \approx \frac{\cos^2(\phi)}{16} \left\{ 4nx - \frac{x^2}{2} - \frac{\epsilon_a^2 \epsilon_b^2}{2} n^2 \right\} \equiv y^2 \qquad (78)$$

and therefore

$$\frac{dP}{dzd\phi} \approx \frac{G\mu^2 \epsilon_a^2 \epsilon_b^2}{8} \sum_n (n\kappa_a + \kappa_b) \left(\frac{n^2}{y^2} - \frac{1}{2} \right) J_n^2(y). \qquad (79)$$

Bearing in mind (76) we take

$$\operatorname{sech} \alpha = w = y/n \approx \cos(\phi)\{4x/n - x^2/2n^2\}^{1/2}/4 \qquad (80)$$

20

so that we can write

$$\frac{dP}{dzd\phi} \approx \frac{G\mu^2\epsilon_a^2\epsilon_b^2}{8}\sum_n (n\kappa_a + \kappa_b)\left(\frac{1}{w^2} - \frac{1}{2}\right) J_n^2(nw).\qquad(81)$$

In our case it is easy to see that w has a maximum at $\tilde{n}_m = x/4$ and the value of the function at this peak is $\tilde{w}_m = \cos(\phi)/\sqrt{2}$. This means that w is at most $1/\sqrt{2}$ and thus $\tanh\alpha$ is at least $1/\sqrt{2}$, and we see that (76) is indeed a good approximation.

We will now perform the sum over n. To do the calculation analytically we must first approximate the sum by an integral, which we can do provided the values of the function are sufficiently close to one another in the region in which we are performing the sum. Secondly, we will assume that the largest contribution to the Bessel function comes from a maximum and that we can approximate that integral by multiplying the height at the peak times the width of this peak at the maximum

$$\frac{dP}{dzd\phi} \sim \sum_n J_n^2(nw) \sim \int dn J_n^2(nw) \sim J_{max}^2 \Delta n.\qquad(82)$$

This will be a reasonable approximation provided the function is sufficiently smooth around the maximum. Below we show that this condition is satisfied.

We can take

$$w^2 \approx c^2 \frac{x}{2n}\left(1 - \frac{x}{8n}\right)\qquad(83)$$

where

$$c = \cos\phi/\sqrt{2}\qquad(84)$$

If we let

$$p = \frac{x}{4n} - 1\qquad(85)$$

then

$$w^2 = c^2(1 - p^2)\qquad(86)$$

which obviously has the maximum value c^2 at $p = 0$. To find the maximum of the integrand of (82) we use the very rough approximation

$$\ln J_n(nw) \sim n(\tanh\alpha - \alpha) = n(\sqrt{1 - w^2} - \operatorname{sech}^{-1} w).\qquad(87)$$

Now if the factor of n were not present, we would reason that since $w = \operatorname{sech}\alpha$ is a decreasing function of α, and $\tanh\alpha - \alpha$ is also a decreasing function

21

of α, then the right hand side not counting the n is an increasing function of w, and (87) will have its maximum where w has its maximum. Since the right hand side is negative, and also a decreasing function of n, we can infer that the actual value of n that maximises (87) is actually less than the one that maximises w. We can thus write

$$n = \frac{x}{4(1+p)} = \frac{x}{4(1+\sqrt{1-w^2/c^2})} \tag{88}$$

and

$$\ln J_n(nw) \sim \frac{x}{4} \frac{\sqrt{1-w^2} - \operatorname{sech}^{-1} w}{1+\sqrt{1-w^2/c^2}} \; . \tag{89}$$

Setting the derivative of (89) to zero gives

$$\frac{1 - w^2 - \sqrt{1-w^2}\,\operatorname{sech}^{-1} w}{w^{-1} - w} = -\frac{1 - w^2/c^2 + \sqrt{1-w^2/c^2}}{w/c^2}. \tag{90}$$

This equation can be solved (although not analytically) for w_m, the value of w that maximises (87) for a particular c and from it, using (88), one can obtain n_m, the value of n at the maximum of (89). Note that both values will be in general different from $\tilde{n}_m$ and $\tilde{w}_m$, the maximum of n and w at the maximum of the function w. Figure 7 shows a plot of w_m, the maximum of (87), as a function of $c = \cos(\phi)/\sqrt{2}$. As expected the value of w where (87) is at a maximum is slightly less than the maximum value of w.

In order to estimate the width at the peak we can again use (87). However in this case we can use the width at the maximum of the function w, i.e., at $p = 0$, or $\tilde{n}_m = x/4$, because it is sufficiently close to the maximum of $J_n(nw)$. This simplifies the calculation considerably. From (87) one can see that

$$\frac{d}{d\alpha} \ln J \sim -n \tanh^2 \alpha \tag{91}$$

and we can compute the width using the Taylor expansion

$$\ln J \sim \ln J_0 - \Delta\alpha\, n \tanh^2 \alpha. \tag{92}$$

Taking $\ln J - \ln J_0 \sim -1$ we find

$$\Delta\alpha \sim -\frac{1}{n \tanh^2 \alpha}. \tag{93}$$

We need to express our answer in terms of Δn however so we take

$$\Delta w \sim \frac{\partial w}{\partial \alpha}\Delta\alpha = \operatorname{sech}\alpha \tanh\alpha \Delta\alpha = \frac{\tilde{w}_m}{\tilde{n}_m\sqrt{1-\tilde{w}_m^2}} \tag{94}$$
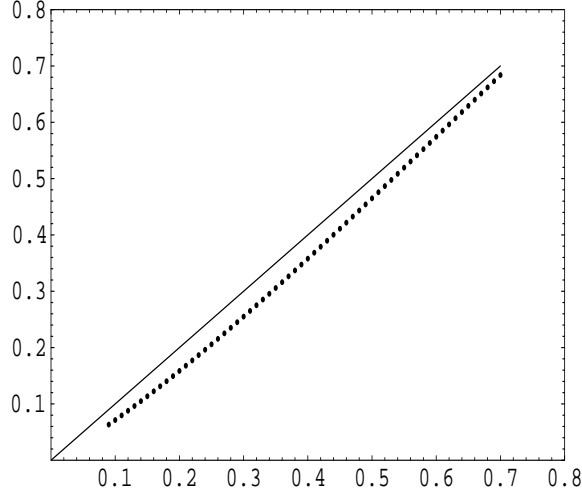
Figure 7: Plot of w_m (points) on the $y-$axis, as a function of $c = \cos(\phi)/\sqrt{2}$. The solid line is $\tilde{w}_m = \cos(\phi)/\sqrt{2}$ for comparison.

and

$$\Delta w = \frac{\partial^2 w}{\partial n^2} \frac{(\Delta n)^2}{2},\tag{95}$$

because the first derivative is 0 at the maximum, yielding

$$\Delta n = \left(\frac{\tilde{n}_m}{\sqrt{1 - \tilde{w}_m^2}}\right)^{1/2} \sim \tilde{n}_m^{1/2}.\tag{96}$$

As advertised above, since $\tilde{n}_m^{1/2} \propto (\epsilon_a^2 \kappa_b / \kappa_a)^{1/2}$ is large in the small κ_a limit, the function around the maximum is sufficiently smooth that we can now perform the sum by approximating it as an integral and further approximating this integral by multiplying the value of the function at the peak by the width near the peak Δn.

For the power per unit length per unit azimuthal angle this procedure yields

$$\frac{dP}{dzd\phi} \sim \frac{G\mu^2\epsilon_a^2\epsilon_b^2}{8}(n_m\kappa_a + \kappa_b)\left(\frac{1}{w_m^2} - \frac{1}{2}\right)J_{n_m}^2(n_m w_m)\,\Delta n\,,\tag{97}$$

where n_m and w_m are given above.

The integral over ϕ can be approximated in an analogous way, taking the value of the Bessel function at the peak, which is at $\phi = 0$, and finding the

23

width at this peak. Evaluating the function at the maximum is straightforward, one simply needs to solve (90) for $\cos\phi = 1$, or equivalently $c = 1/\sqrt{2}$. This procedure yields $w_0 \approx .691584$ and from (88), $n_0 \approx x/4.83352$.

In this case the width can be found by using $\ln J \sim f(w_m)$ as given by (89) and we can see that

$$\frac{d^2 f(w_m)}{d\phi^2} = \frac{df}{dw_m}\frac{d^2 w_m}{d\phi^2} \tag{98}$$

because $dw_m/d\phi = 0$. Using $\ln J - \ln J_0 \sim -1$ again yields

$$\frac{(\Delta\phi)^2}{2}\frac{df}{dw_m}\frac{d^2 w_m}{d\phi^2} \sim -1 \tag{99}$$

and because $f \propto x$, ignoring factors of $O(1)$ we have that

$$\Delta\phi \sim x^{-1/2}. \tag{100}$$

These considerations yield for the power per unit length

$$\begin{aligned}
\frac{dP}{dz} &\sim G\mu^2 \epsilon_a^2 \epsilon_b^2 (n_0\kappa_a + \kappa_b) J_{n_0}^2 (n_0 w_0)\, \Delta n \Delta\phi \\
&\sim G\mu^2 \epsilon_a^2 \epsilon_b^2 (n_0\kappa_a + \kappa_b) J_{n_0}^2 (n_0 w_0)
\end{aligned} \tag{101}$$

Expressing this equation instead using (76), with inverse hyperbolic functions written out explicitly, and ignoring factors of $O(1)$ yields

$$\frac{dP}{dz} \sim G\mu^2 \epsilon_b^2 \kappa_a \left(\frac{w_0 e^{\sqrt{1-w_0^2}}}{1 + \sqrt{1-w_0^2}} \right)^{2n_0} \tag{102}$$

which can be written more intelligibly as

$$\frac{dP}{dz} \sim G\mu^2 \epsilon_b^2 \kappa_a e^{-\alpha x} \tag{103}$$

where $\alpha \approx 0.07867$ can be calculated from w_0 and n_0.

References

[1] T.W.B. Kibble, J. Phys. A9 (1976) 1387.

[2] A. Vilenkin and E.P.S Shellard, Cosmic strings and other Topological Defects. Cambridge University Press, 2000.

[3] D.P. Bennet and F.R. Bouchet, Phys. Rev. Lett. 60 (1988) 257.

[4] V. Berezinski, B. Hnatyk and A. Vilenkin, astro-ph/0102366.

[5] T. Damour and A. Vilenkin, Phys. Rev. Lett. 85 (2000) 3761.

[6] P. Bhattacharjee and G. Sigl, Phys. Rep. 327 (2000) 109.

[7] M. Sakellariadou, Phys. Rev D42 (1990) 354.

[8] M. Hindmarsh, Phys. Lett. B251 (1990) 28.

[9] D. Garfinkle and T. Vachaspati, Phys. Rev. D42 (1990) 1960.

[10] S. Weinberg, Gravitation and Cosmology. John Wiley and Sons, 1972.

[11] G.N. Watson, A Treatise on the Theory of Bessel Functions. Cambridge University Press, 1922.

[12] R.A. Battye and E.P.S.Shellard, Nucl. Phys. B423 (1994) 260.

[13] X. Martin, Phys. Rev. D53 (1996) 6847.

[14] J.J. Blanco-Pillado and K.D. Olum, Phys. Rev. D59 (1999) 063508.

[15] B. Carter, Phys. Rev. D41 (1990) 3869.; A. Vilenkin, Phys. Rev. D41 (1990) 3038

[16] K.D. Olum, J.J. Blanco-Pillado and X. Siemens, Nucl. Phys. B599 (2001) 446.